NOTE ON THE nth DERIVATIVE OF A DETERMINANT WHOSE CONSTITUENTS ARE FUNCTIONS OF A GIVEN VARIABLE.*

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Let the determinant be $D=(a_1 b_2 \ldots\ldots l_r)$ where the a, b, ..., l are functions of a variable t. Then

$$\frac{dD}{dt}=(a_1{}' \, b_2 \ldots\ldots l_r)+\ldots\ldots+(a_1 \, b_2 \ldots\ldots l_r{}').$$

$$\frac{d^2 D}{dt^2}=(a_1{}'' \, b_2 \ldots\ldots l_r)+\ldots\ldots+(a_1 \, b_2 \ldots\ldots l_r{}'')$$

$$+\,2(a_1{}' \, b_2{}' \, c_3 \ldots\ldots l_r)+\ldots\ldots+2(a_1 \, b_2 \ldots\ldots k'_{r-1} \, l_r{}').$$

Consider now the expansion

$$(a+b+c+\ldots\ldots+l)^2=a^2 \, b^0 \, c^0 \ldots\ldots l^0+a^0 \, b^2 \, c^0 \ldots\ldots l^0+2a^1 \, b^1 \, c^0 \ldots\ldots l^0+\ldots\ldots$$

If we interpret the power of a^n as the nth derivative of a and write instead of $a^2 \, b^0 \, c^0 \ldots\ldots l^0$ the expression $(a_1{}'' \, b_2 \, c_3 \ldots\ldots l_r)$, etc., we have, symbolically,

$$\frac{d^2 D}{dt^2}=(a+b+c+\ldots\ldots+l)^2.$$

Suppose now that, symbolically,

$$\frac{d^{n-1} D}{dt^{n-1}}=(a+b+c+\ldots\ldots+l)^{n-1}=\sum\frac{(n-1)\,!}{\alpha\,!\,\beta\,!\ldots\ldots\lambda\,!}a^\alpha \, b^\beta \ldots\ldots l^\lambda\,,$$

where $\alpha+\beta+\gamma+\ldots\ldots+\lambda=n-1$.

Now suppose the symbolic form be interpreted as before and another differentiation with respect to t carried out. Then the coefficient of $a^{\alpha'} \, b^{\beta'} \ldots l^{\lambda}$ will be

$$C=\frac{n\,!}{\alpha'\,!\,\beta'\,!\ldots\ldots\lambda'\,!},$$

where $\alpha'+\beta'+\ldots\ldots+\lambda'=n$. For this term can be obtained by differentiation from terms with exponents one less than α', β', ..., or λ', and its coefficient will be

$$\frac{(n-1)\,!}{(\alpha'-1)\,!\,\beta'\,!\ldots\ldots\lambda'\,!}+\frac{(n-1)\,!}{\alpha'\,!\,(\beta'-1)\,!\ldots\ldots\lambda'\,!}+\ldots\ldots=C.$$

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